

PRESENTING: α SUPERCONFORMAL FLAVOR SIMPLIFIED ; D. POLAND & D. SIMMONS-DUFFIN
arXiv: 0910.4585 (hep-ph)

[NOTICE: IN LIGHT OF ISSUES THIS SEMESTER WITH TALK LENGTHS, RESTLESSNESS, & ATTENTION SPAN, I AM PREPARING A SHORT TALK. [EVEN WITH DISCUSSION, I DON'T INTEND TO USE THE WHOLE 1.5 HOUR.] ... I ENCOURAGE OTHERS TO CONSIDER THIS AS WELL.]

WE WILL ONLY FOCUS ON GENERAL TOOLS & PRINCIPLES THAT CAN BE LEARNED FROM THIS PAPER! (§2, maybe §3.1)

↳ I THINK THIS IS WHAT IS APPROPRIATE FOR A JOURNAL CLUB TALK. "REALISTIC MODELS" ARE 'DETAILS' & TEND TO BE LESS INTERESTING FOR THOSE WHO ARE NOT ACTIVELY WORKING ON SIMILAR MODELS ALREADY.

I ENCOURAGE BROAD DISCUSSION, BUT I DO NOT HAVE MANY ANSWERS.

Refs.

* 0910.4585: THIS PAPER. ~~#####~~ WE ARE ONLY LOOKING @ P. 4-10 ... IF YOU'RE SITTING @ MY TALK AND ARE JUST LOOKING AT THE PAPER RIGHT NOW, OR PERHAPS YOU DIDN'T EVEN PRINT THE PAPER BUT JUST PRINTED MY NOTES ... THEN I AM VERY DISAPPOINTED IN YOU. ; IT'S OK. WE CAN STILL BE FRIENDS.

• <http://github.com/davidsd/lie>

↳ THIS IS DAVID OD'S PYTHON CODE FOR WORKING W/ LIE GROUP REPRESENTATIONS, BASED ON LIE BY VAN LEEUWEN, WEN, & LISSER ('90s).

WE WILL NOT DISCUSS THIS, BUT IT IS A BROADLY APPLICABLE MODEL BUILDING TOOL THAT I THINK IS WORTH MENTIONING.

• hep-th/0304128: THE ORIGINAL α -MAXIMIZATION PAPER, see also follow ups. RENNY SST ~ 2003.

↳ OTHER GOOD REFS INCLUDE WIECHT'S CAMBRIDGE TALKS, TEENINGS BOOK.

• hep-th/2007240: REVIEW OF SOFT AS OF 2000 (BEFORE α -MAXIMIZATION!)

• hep-ph/000625: ORIGINAL "SUPERCONFORMAL FLAVOR" PAPER. (SEE ALSO REFS THEREIN FOR ORIGINAL ANAHEHY REFS.)

↳ X₃₂₁ STRONG SECTORS. (PRESENT PAPER USES VECTOR)

FORTUNATELY, INTRILIGATOR & WECHT PROVIDE US WITH A TRICK:

PARAMETERIZE A TRIAL (ie arbitrary) R-CHARGE AS

$$R_t = R_0 + \sum_I s_I F_I$$

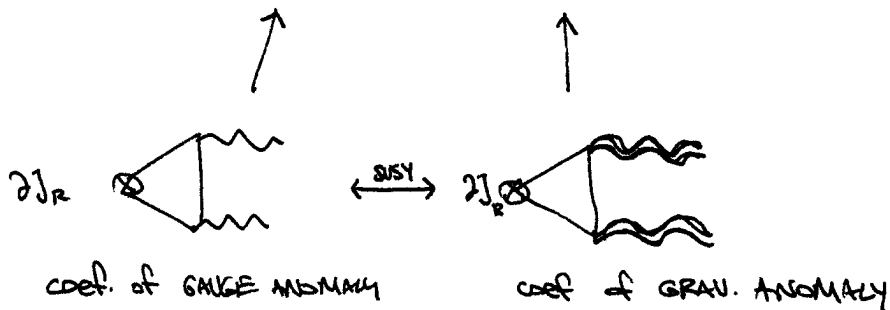
↑
ACTUAL SCFT
R-CHARGE WE
WANT.

↑
PARAMETERS

↑
GENERATOR OF U(1) SYMMETRY
ASSOCIATED WITH THE Ith FLAVOR
GROUP OF THE THEORY.

THEN R_0 CAN BE PICKED OUT BY MINIMIZING $a(R_t)$ OVER s_I :

$$a(R_t) = \frac{3}{32} [3 \text{Tr}(R_t^3) - \text{Tr}(R_t)] \quad (\text{trace over fields})$$



ie. $\min_{R_t} a(R_t) = a(R_0)$
 ↑
 ↑ is over $\{s_I\}$

[This is for FUP'S
OWN NOTES. NOT FOR
JC TALK.]

DISCUSSION [skip this] see: Wecht

$$\langle T_r^r \rangle = a(\text{Euler}) + c(\text{Weyl}) + \dots$$

↑
JUST WRITING w/ GEOMETRIC QUANTITIES.

CARDY'S q-"thm": $a_{IR} \langle a_{UV} \rangle$

↑
cf c-thm in 2D ; c is AN IMPORTANT OBJECT THAT COUNTS DOF.

IN A SCFT, CAN COMPUTE EXACT RESULTS FROM 't HOOFT ANOMALIES (AFGJ) '97

$$a = \frac{3}{32} (3 \text{Tr} R^3 - \text{Tr} R)$$

$$c = \frac{1}{32} (9 \text{Tr} R^5 - 5 \text{Tr} R)$$

This comes from $ST = a(S\text{Euler}) + c(S\text{Weyl})$

CONTAINS $\partial R, T_r^r$

BOTH CONTAIN $[Riem], [Riem], F_R \tilde{F}_R$

$$\text{get: } \partial R = \underbrace{(\#a + \#c)}_{\text{Tr } U(1)_R} Riem \tilde{Riem} + \underbrace{(\#\tilde{a} + \#\tilde{c})}_{\text{Tr } U(1)_{R^2}} F_R \tilde{F}_R$$

SO CAN INVERT UN OMB. TO WRITE a & c.

~~ANOMALY FREE CONDITION (SUSY QFT)~~

SOME QUICK EXAMPLES (Following Wight)

1. SQC : $SU(N_c)$ w/ N_f VECTORLIKE FIELDS Q, \tilde{Q}

$$ANOMALY = \sum_i q_i T(C_i) = 2N_c + 2N_f (R[Q] - 1)$$

\uparrow GAUGINOS IN ADJ \uparrow FERMION'S R CHARGE
 note $R(Q) = R(\tilde{Q})$ so WE JUST DOUBLED THIS.

ANOMALY FREE $\Rightarrow R[Q] = R[\tilde{Q}] = 1 - N_c/N_f$ ✓

NO NEED TO α -MAXIMIZE.

✓ Very simple deformation.

2. SQC DEFORMED w/ ADJOINT MATTER, X ($N_{free} = 0$)

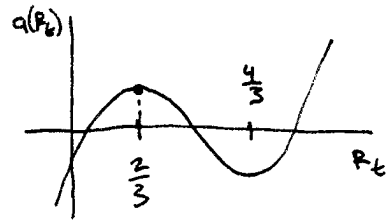
$$ANOMALY = \underbrace{2N_c + 2N_f (R[Q] - 1)}_{SQC} + \underbrace{2N_c (R[X] - 1)}_{ADJOINT \psi_X \text{ R CHARGE}}$$

ANOMALY FREE CONDITION NOW HAS 1 PARAMETER FAMILY!
 ONE EQN, 2 UNKNOWNNS $R[Q], R[X]$.

[We won't use a max on this. Instead, let's do a simpler example]

3. FREE YSF Φ

$$a(R_\pm) = 3(R_\pm - 1)^3 - (R_\pm - 1)$$



$$a(R_\pm)|_{max} = a(R_0) \Rightarrow \boxed{R_0 = 2/3}$$
 ✓

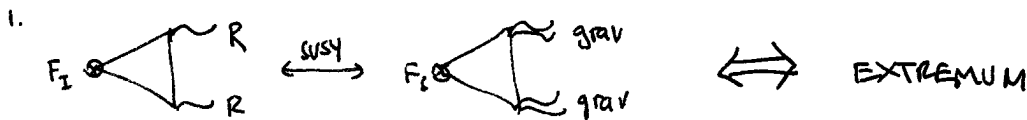
ONE WOULD HAVE TO DO THIS FOR THE R CHARGE OF EACH FIELD IN THE THEORY. THIS IS USUALLY DONE NUMERICALLY.

DISCUSSION [SKIP THIS - FOR FLIP'S NOTES ONLY]

Motivation of proof

$$\max a \Rightarrow \begin{cases} \text{1. EXTREMUM: } 9 \operatorname{Tr} R_{ij}^2 F_i = \operatorname{Tr} F_i \\ \text{2. MAXIMUM: } \operatorname{Tr} R_{ij} F_i F_j < 0 \end{cases} \quad \begin{matrix} \partial a / \partial S_i = 0 \\ \partial a / \partial S_{ij} < 0 \end{matrix}$$

sanity check: free theory has $R_0 = 2/3$.



2. CONFORMAL INVARIANCE: $\langle R F_i F_j \rangle \sim \langle F_i F_j \rangle$ (non-trivial)

POSITIVE DEF BY INTUITION
OR, HOMOGRAPHICALLY TURNS INTO
GAUGE CURRENTS IN BULK SO THAT
THIS IS KIN. TERM W/ DEF SIGN.

IF FREE THY: $\langle R F_i F_j \rangle = -\frac{1}{3} \langle F_i F_j \rangle$

re bc $\langle F_i F_j \rangle > 0 \Rightarrow \langle R F_i F_j \rangle < 0. \Rightarrow$ MAXIMUM.

IR ACCIDENTAL SYMMETRIES COMPlicate THINGS "□"

... there are some subtleties. (see, eg § 2.3) we won't discuss further.

ok, good. so now we've met a -MAXIMIZATION & WE KNOW WHAT IT'S GOOD FOR (GIVING R_0). BUT WHAT IS IT REALLY GOOD FOR?

ie: SO WHAT?

Some past applications

- PROGRESS TOWARD 4D ZAMOLDCHIKOV C-THM
- INDIRECT PROOF (CHECK) OF SEIBERG DUALITY
- PROGRESS TOWARD PROOF OF ADS/CFT
- BRANE THINGS IN STRING THY

⋮

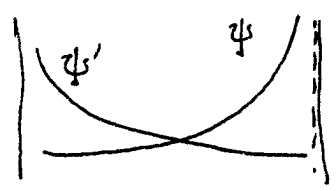
~~GEN~~ GEN IDEA: PROBE NONREPUGNATIVE REGION @ (near) SUPERCONF. FIXED PT.

↳ pheno/model-building games we can play w/ strong coupling.

"MOTIVATION & INTRO" PT II ← most of this talk is motivation & intro

OUR GROUP HAS A LOT OF EXPERIENCE W/ STRONG COUPLING: AS
IN FACT, ONE TOPIC THAT WE'VE EXPLORED RECENTLY ARE
ANARCHIC FLAVOR MODELS. (DC, YT, FT)

→ IDEA: FLAVOR HIERARCHIES IN YUKAWA SECTOR: WTF?
NATURALNESS ⇒ $\mathcal{O}(1)$ DIM'LESS PARAM.
↓
AS PROVIDES A WAY TO TURN SMALL #'S INTO BIG #'S BY WARPING.
ORIGINALLY USE WARPING FOR HIERARCHY PROBLEM (HIGGS),
BUT NOW PIGGY BACK FLAVOR HIERARCHY AS "ADDED BENEFIT"



MAKE ^{4D} YUKAWAS EXPONENTIALLY SMALL BY HAVING A SMALL SD
WAVEFUNCTION OVERLAP W/ ANARCHIC $\mathcal{O}(1)$ YUKAWA COUPLINGS.

THIS WAVEFUNCTION LOCALIZATION IS CONTROLLED BY THE BULK
MASS PARAMETER, $c \equiv RM \sim \mathcal{O}(1)$. SMALL ~~DIFFERENCES~~
DIFFERENCES IN c GIVE BIG HIERARCHIES.

→ these parameters are INPUTS to the model.
NATURAL, BUT AN ADDITIONAL SOURCE OF DOFS. ∴

↳ this whole picture is dual to a strongly coupled theory ^{outputs of a theory}
VIA THE ADS/CFT CORRESPONDENCE.

IN THIS PICTURE, THE c PARAMETERS CORRESPOND TO ANOMALOUS DIMENSIONS

↳ MOTIVATES A DIFFERENT PERSPECTIVE: START FROM THE CFT SIDE
AND CONSIDER A STRONGLY COUPLED THY THAT PREDICTS THE
APPROPRIATE ANOMALOUS DIMENSIONS.

... OF COURSE STRONGLY-COUPLED THEORIES ARE HARD TO CALCULATE IN,
SO WE SHOULD GO TO A REGIME WHERE WE HAVE SOME TRICKS;
SUPERCONFORMAL THEORY.

IDEA: WE STILL HAVE ANARCHY: YUKAWAS & ANOMALOUS DIM ARE $\mathcal{O}(1)$
BUT RG FLOW GENERATES FLAVOR HIERARCHIES.

ORIGINALLY PROPOSED BY NELSON & STRASSLER (hep-ph/0006251)
BUT DETERMINING ANOMALOUS DIM IN SCFT ↔ DETERMINE R CHARGES.

↳ so DP & DSD USED Q-MAXIMIZATION TO WORK OUT PREVIOUSLY
"INCALCULABLE" MODELS.

SINCE WE'LL BE INTRODUCING AN ADDITIONAL (STRONGLY COUPLED) SECTOR, THESE MODELS TEND TO INTRODUCE LOTS OF NEW FIELDS.

→ WANT "FLAVOR PHYSICS" @ HIGH SCALE (eg near M_{Pl}) SINCE

- AND/OR → 1. WORRY ABOUT PERTURBABILITY OF SM GAUGE COUPLINGS
 2. WORRY ABOUT BARYON # & LEPTON # OPERATORS } "EASY FLAVOR PROBLEM"

ASSUME CONFORMAL FLAVOR SECTOR LIVES ABOVE M_{GUT}

in fact, these kinds of models shed new light on subtleties in the EASY FLAVOR PROBLEM.

SO DP & DSD LOOK @ VARIANCE $SU(5)$ GUT PAPS. WE WON'T GO INTO THEIR 'REALISTIC MODELS'

→ BUT RECALL TYPICAL SM EMBEDDING INTO $SU(5)$

$$T_i = \begin{pmatrix} e \\ -\nu \\ d_1 \\ d_2 \\ d_3 \end{pmatrix} \left. \begin{array}{l} \} L \\ \} d_R \end{array} \right\} \bar{5}$$

$$T_i = \begin{pmatrix} 0 & \bar{e}_R & u_1 & u_2 & u_3 \\ 0 & 0 & d_1 & d_2 & d_3 \\ 0 & 0 & 0 & \bar{u}_3 & -\bar{u}_2 \\ 0 & 0 & 0 & 0 & \bar{u}_1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \left. \begin{array}{l} \} Q \\ \} 10 \end{array} \right\} 10$$

↑ ALSO HES
HES

↑ DP & DSD CONSIDER "10-COUPLED MODELS" WHERE SUPPRESSION IS ATTACHED TO T_i 's.

DP & DSD DISCUSS PATTERNS IN THE CKM & MNS MATRICES THAT SHOW THAT WAVEFUNCTION RENORMALIZATION OF F, T, H COULD PLAGIARIS GENERATE THE SM FLAVOR STRUCTURE; § 2.1.

RG? BUT YUKAWA CW ? W IS NOT RENORMALIZED! HOWEVER, THE KÄHLER POT IS RENORMALIZED; DYNAMICAL ORIGIN OF FLAVOR HIERARCHY.

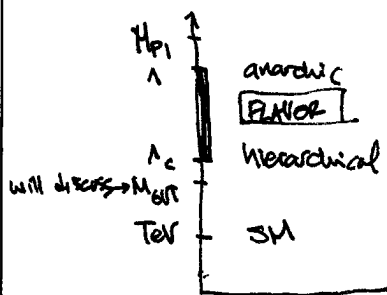
Lightning review

$$\mathcal{L} = \int d^4x \sum_i Z_i \Phi_i^\dagger \Phi_i$$

s.t. CANONICAL NORMALIZATION GIVES SUPPRESSION $\epsilon_i = Z_i^{-1/2}$ FOR EACH FIELD IN AN OPERATOR

$$\frac{d \log Z_i}{d \log \mu} = -\gamma_i \Rightarrow \epsilon_{\Phi_i} = e^{-\frac{1}{2} \int_{\log \lambda}^{\log \mu} \gamma_i d \log \mu}$$

Λ : UV scale @ WHICH YUKAWAS ANARCHIC
 Λ_c : FLAVOR SECTOR DECOUPLES



APPROXIMATE γ VIA: IGNORE \mathcal{L}_{SM} , YUKAWAS
 ↑ TREAT THY AS SOFT W/ STRONG GAUGE GROUP G .
 ⇒ γ CONST,

$$\epsilon_{\Phi_i} = \left(\frac{\Lambda_c}{\Lambda}\right)^{\frac{1}{2} \gamma_i}$$

↑ want $O(1)$ γ 's, so LARGE RG EVOL FROM $\Lambda \rightarrow \Lambda_c$.

FACT: IN THE PHYSICAL BASIS WHERE FIELDS ARE CANONICALLY NORMALIZED @ ea M, FOR $W = \dots + c \Theta + \dots$

$$\frac{\partial c}{\partial \log M} \equiv \beta_c = c(d_{\text{dim}} \Theta - 3)$$

SM FIELDS, NOT CHARGED UNDER STRONGLY COUPLED FLAVOR SECTOR, G.

for a YUKAWA COUPLING $W = y FTH$ or $y TTH$

$$\frac{\partial y}{\partial \log M} = y \left(3 + \sum \frac{Y_i}{2} - 3 \right) = \frac{y}{2} (Y_{FT} + Y_T + Y_H)$$

$$\Rightarrow Y_{u,d}^{ij}(\Lambda_c) = \left(\frac{\Lambda_c}{\Lambda} \right)^{\frac{1}{2} \sum Y} y_{u,d}^{ij}(\Lambda)$$

(plus is the flavor structure that is frozen in)

~~TOY MODEL~~

OUR GOAL

DONE BY NELSON + SCHARNER, BUT THEIR CALCULABLE MODELS NEEDED MANY FIELDS + INTERACTIONS ... we'll use a-max

ANARCHY IN UV YUKAWAS
(1) ANOMALOUS DIM

INPUT "STRUCTURE" IN RS MODELS IS NOW AN OUTPUT

BUT: LET REPRESENTATIONS GIVE US THE SCALING THAT EXPONENTIATES INTO FLAVOR HIERARCHIES.

TOY MODEL (§ 2.2.1)

	SU(5) _{OUT}	G = SU(N)
$X + S$	$10 + 1$	\square
$\bar{X} + \bar{S}$	$\bar{10} + 1$	$\bar{\square}$

↑ M CFT regime
CFT sees SU(5) RGD w/ 11 flavors; CFT fixed point @ $\frac{5}{2} N < 11 (3N)$

DEFORMATION $\Delta W \equiv h(M) \mu^{3-d} \Theta$

$d_{\text{dim}}[\Theta] = d$ ("ENGINEERING" DIMENSION)
relevant operator → GROWS IN IR

As $M \rightarrow IR$, this op. GROWS UNTIL $B_h \approx 0$ @ FIXED ^{NEW} POINT

$$B_h = (d_{\text{dim}} \Theta - 3) h = (d + Y_{\Theta}/2 - 3) h$$

↑ $d_{\text{dim}} \Theta \approx 3$, OP BECOMES MARGINAL, $h(M) \approx h_*$

SO WE'RE STARTED @ UV SCALE Λ , DEFINED Λ_c AS $h(\Lambda_c) = h_*$.

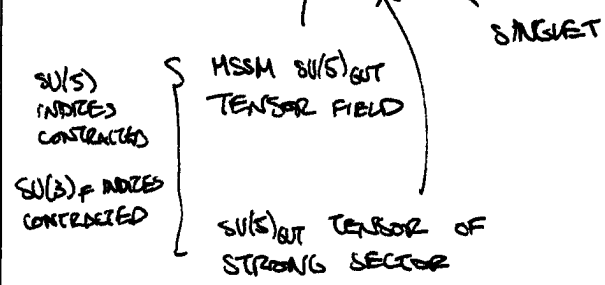
WHAT Θ 's ARE PERMITTED?

THE 4D MODEL ADMITS ONLY ONE RELEVANT GAUGE INT COUPLING TO MSSM.

↑
 irrelevant ones die
 over $\Lambda \rightarrow \Lambda_c$ running

↑
 WE "WEAKLY GAUGE"
 MSSM GROUP.

$$W_{int} = h [T, X] S$$



THE "1" IN T_1 INDEXES A FLAVOR COMBINATION. WE WILL MAKE THIS GENERATION LIGHT.

IN THIS 4D MODEL WE TREAT h AS AN SU(3)_F SELECTION THAT PICKS OUT A T_i .

↑
defines

GOOD. NOW WE JUST HAVE TO DETERMINE γ 's OF SM FIELDS (γ_{T_i})

$$\alpha(\vec{R}_t) = \frac{3}{32} \left[2(N^2 - 1) + \sum_i \dim(r_i) (3(R_i - 1) - (R_i - 1)) \right]$$

↑
 $(R_x, R_{\bar{x}}, R_s, R_{\bar{s}}, R_t)$

↑
 T_2, T_3 ARE LFT SINGLETS
 $\Rightarrow R = (2/3)$, i.e. free

ADDITIONAL CONSTRAINTS

1. ANOMALY FREE W/RT $G = SU(N) \Rightarrow D = T(G) + \sum (R_i - 1) T(r_i)$

↳ remark: this just gives $\beta_{g_g}^{NSV2} = 0$; i.e. theory = conformal.

2. T, X MUST BE MARGINAL w/rt G

↳ dim = 3; ~~dim = 3; $\beta_{g_g}^{NSV2} = 0$~~

in W

BUT @ SOFT FIXED POINT, $D = \frac{3}{2}R$

$\Rightarrow \frac{3}{2}R = 2 = R_t + R_{\bar{x}} + R_s$

DOING THIS ~~MAX~~ g -MAXIMIZATION, WE OBTAIN TABLE 2. THE RELEVANT RESULT

R_{T_1}	$N=4$	$N=5$	$N=6$	$N=7$
	.69	.77	.92	1.19

← soft for $\frac{3}{2}N < N_f < 3N$

~~typically $D < \frac{3}{2}N$~~

AS N BIGGER, MORE STRONGLY COUPLED, LARGER R_{T_1} .

$G_{T_1} = \left(\frac{\Lambda_c}{\Lambda}\right)^{\frac{3}{2}R_{T_1} - 1} \Rightarrow$ LARGER $R_{T_1} \Rightarrow$ SMALLER CONFORMAL WINDOW.

THAT'S IT. I WANT TO CLOSE (if there's time) WITH SOME REMARKS ABOUT DP & DSD'S VECTORLIKE MODELS

- NELSON & STRASSER: XFL MODELS: Λ_c GEN DYNAMICALLY BY EXTRA GAUGE GP.
- DP+PSD: VECTORLIKE MODELS \rightarrow SIMPLEST

↓
 SIMPLEST OPT EXIT: JUST ~~AS~~ INTRODUCE A MASS TERM.
 eg. $W = m_x X X \rightarrow \Lambda_c = m_x \left(\frac{\Lambda_c}{\Lambda}\right)^2 (\chi_x + \bar{\chi}_x)$

- SIMPLEST MODIF OF TOY MODEL: "10-CENTERED" MODEL WHERE T_1 & T_2 GET LARGE X

$$W_{int} = T_1 \Theta_1 + T_2 \Theta_2 \quad \left(\begin{array}{ccc} & SU(5)_{GUT} & G = SU(N) \\ X + \bar{2} + S & 10 + \bar{5} + 1 & \square \\ \bar{X} + 2 + \bar{S} & \bar{10} + 5 + 1 & \bar{\square} \end{array} \right)$$

\uparrow \uparrow
 XQ $\bar{X}S$

exhausts OPS. (ORIGINAL, MSSM COUPLING, GAUGE INT.)

- IN SIMPLEST MODELS, Θ_1 & Θ_2 VIOLATE LEPTON + BARYON # SO @ Λ_c . INTEGRATING OUT CFC SECTOR \rightarrow P^+ DECAY OPS. (dim 6)

↳ SUPPRESSION IS $\frac{g^2 T^2}{M \Lambda_c^2}$

\Rightarrow REQ. ~~AS~~ $M_{GUT} \leq \Lambda_c < \Lambda \leq M_{Pl} \Rightarrow$ SMALL CONF WINDOW

\Rightarrow FAIRLY STRONG COUPLED

Turns out that LANDAU POLE IS TOO STRONG IN "SIMPLEST-EST" MODELS LARGEST ALLOWED N'S IN CONFORMAL RANGE STILL GIVE LANDAU POLES.

SUMMARY

1. α -MAXIMIZATION: determine $U(1)_R$ OF SCFT
2. CAN USE STRONG DYNAMICS TO GEN FLAVOR HIERARCHIES
 - Y 'S ARE AN OUTPUT, NOT AN INPUT
3. TENSION: eg. LANDAU POLE VS. P^+ DECAY.