Review of D-$\overline{D}$ Mixing

David Curtin

Cornell University

Cornell Institute for High Energy Phenomenology

A-Exam Presentation

Wednesday, October 7 2009

Introduction

- Neutral meson mixing probes the deep quantum structure of the theory.
- Can reveal CP Violation: interesting for Baryo/Leptogenesis.
- D-$\overline{D}$-mixing only example of meson oscillation in the up sector.
  - CPV is tiny in SM but hard to calculate:
    $\longrightarrow$ BSM search rather than precision test!
  - has only recently been discovered!

Outline:

- Formalism
- SM calculations
- Experimental Measurements

Formalism

Weak Basis

- Produce a weak eigenstate D^0 or $\overline{D}^0$. Time evolution given by Schrödinger Eqn

$$i\frac{\partial}{\partial t}\begin{pmatrix}\overline{D}^0\\ D^0\end{pmatrix} = \mathbf{H}\cdot\begin{pmatrix}\overline{D}^0\\ D^0\end{pmatrix}, \quad \text{where} \quad \mathbf{H} = \mathbf{M} - \frac{i}{2}\Gamma.$$

  H is not hermitian due to decays.

- How do we get **H** from the underlying theory?

$$\left(\mathbf{M} - \frac{i}{2}\Gamma\right)_{ij} = \frac{1}{2m_D}\langle D_i|H_{\text{eff}}|D_j\rangle =$$

$$m_D^{(0)}\delta_{ij} + \frac{\langle D_i|H_w|D_j\rangle}{2m_D} + \frac{1}{2m_D}\sum_f \frac{\langle D_i|H_w|f\rangle\langle f|H_w|D_j\rangle}{m_D^{(0)} - E_f + i\epsilon}$$

- To solve time evolution go to mass basis. **H** has eigenvalues $\omega_{L,H}$ and eigenstates

$$|M_{L,H}\rangle = p|M^0\rangle \pm q|\overline{M}^0\rangle.$$

- $m = \mathrm{Re}\,\omega$ and $\Gamma = -2\mathrm{Im}\,\omega$. Define

$$\Delta m = m_H - m_L \qquad \Delta\Gamma = \Gamma_H - \Gamma_L$$

and

$$x \equiv \frac{\Delta m}{\Gamma}, \quad y \equiv \frac{\Delta\Gamma}{2\Gamma} \quad \Rightarrow \quad (x+iy)\Gamma = \frac{\langle D|H_{\mathrm{eff}}|\overline{D}\rangle}{m_D}$$

to describe mixing.

Formalism
Time-Dependent Decay Rate

- An initially pure weak eigenstate $M, \overline{M}$ oscillates with time:

$$\begin{aligned} |M^0_{\mathrm{phys}}(t)\rangle &= g_+(t)|M^0\rangle - \frac{q}{p}\, g_-(t)|\overline{M}^0\rangle \\ |\overline{M}^0_{\mathrm{phys}}(t)\rangle &= g_+(t)|\overline{M}^0\rangle - \frac{p}{q}\, g_-(t)|M^0\rangle \\ \text{where} \quad g_\pm(t) &= \frac{1}{2}\left(e^{-im_H t - \frac{1}{2}\Gamma_H t} \pm e^{-im_L t - \frac{1}{2}\Gamma_L t}\right). \end{aligned}$$

- If $D^0 \to f$ is forbidden/suppressed, call it the wrong sign decay. Rate is given by

$$r(t) = \frac{|\langle f|D^0_{\mathrm{phys}}(t)\rangle|^2}{|\overline{A}_f|^2} = \left|\frac{q}{p}\right|^2 \left|g_+(t)\lambda_f^{-1} + g_-(t)\right|^2$$

where we have normalized to the right-sign amplitude to **eliminate hadronic junk**.

Having complex phases in your Lagrangian is a sure way to get CPV.

But there are two types of phases that can appear in amplitudes:

- **weak phases** ϕ which appear in the Lagrangian directly. They are opposite for A_f, $\overline{A}_{\bar{f}}$.
- **strong phases** δ due to intermediate on-shell states in the decay process. They are due to CP-conserving interactions (mostly QCD) and are the same for A_f, $\overline{A}_{\bar{f}}$.

Only weak phases give CPV!

Often $\phi = \arg(q/p)$ is just called the "weak phase".

Can classify CPV using two criteria.

From importance of strong phases:

- **indirect CPV:** only weak phases
- **direct CPV:** both strong and weak phases

Based on where in the decay rate expression CPV occurs:

(decay rate)^2 $=$ (direct)^2 $+$ (via mix)^2 $+$ (direct)(via mix)

1. CPV in decay: $|\overline{A}_{\overline{f}}/A_f| \neq 1$
2. CPV in mixing: $|q/p| \neq 1$
3. CPV in interference: $\mathrm{Im}\,\lambda_f = \mathrm{Im}\left(\frac{q}{p}\frac{\overline{A}_f}{A_f}\right) \neq 0$

Estimation of D-$\overline{D}$-mixing in SM

GIM Mechanism

- In the SM, tree-level FCNC's are forbidden by GIM mechanism

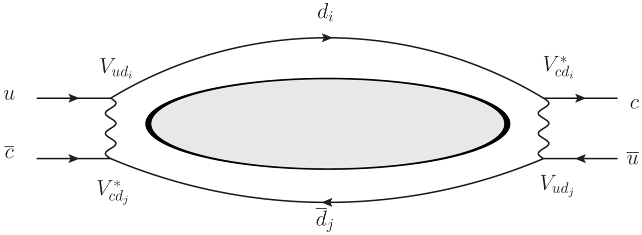
$\Rightarrow$ Meson mixing is loop-suppressed!

- Only flavor violation in V_{CKM} matrix, which is highly hierarchical.

$\Rightarrow$ 3rd gen contribution to mixing is small, can treat D-mixing with 2 generations.

  $\rightarrow$ almost no CPV in SM prediction, any signal $\gtrsim 10^{-3}$ is NP!

- 2 generations: $V_{\rm CKM} \to \begin{pmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{pmatrix}$
- Can ignore CPV



$\propto V_{ud_i} V^*_{cd_i} V^*_{cd_j} V_{ud_j}$
$\downarrow$
$\pm \sin^2\theta_c \cos^2\theta_c$

- in 2-gen $SU(3)$ limit, the $d_i = d, s$ are identical!

$\Rightarrow$ can collect mixing contributions into groups that each give net zero contribution

  - members are identical up to a sign and cancel!

$\Rightarrow$ Mixing is an $SU(3)$-breaking effect.

The effective 4-fermi vertex due to the SM box diagram is

$$\mathcal{L}^{\Delta c=2} = \frac{G_F^2}{8\pi^2} V_{cs}^* V_{us} V_{cd}^* V_{ud} \frac{(m_s^2 - m_d^2)^2}{m_c^2} (\mathcal{O} + 2\mathcal{O}')$$

$$\mathcal{O} = \overline{u}\gamma^\mu(1+\gamma_5)c\,\overline{u}\gamma_\mu(1+\gamma_5)c, \qquad \mathcal{O}' = \overline{u}(1-\gamma_5)c\,\overline{u}(1-\gamma_5)c$$

Since the c is much heavier than the s, this is much smaller than the corresponding Kaon diagram.

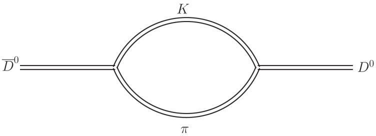
We get

$$\Delta m_D^{\mathrm{box}} = \frac{\langle D|H_w|D\rangle}{m_D} \sim 10^{-18}\,\mathrm{GeV}$$

From Datta, Kumbhakar 1985

Estimation of D-$\overline{D}$-mixing in SM

Long-Range Contributions



$$i\mathcal{M} \sim A(g)\log\left(-p^2/\Lambda^2_{\rm QCD}\right) = A(g)\log\left(p^2/\Lambda^2_{\rm QCD}\right) + i\pi A(g)$$

$$\text{Optical Theorem} \rightarrow m\Gamma = \pi A(g)$$

$$\Delta m = \mathrm{Re}\,\frac{\langle\overline{D}|H|D\rangle}{m_D} = \frac{\Gamma}{\pi}\log\frac{m_D^2}{\Lambda^2_{\rm QCD}},$$

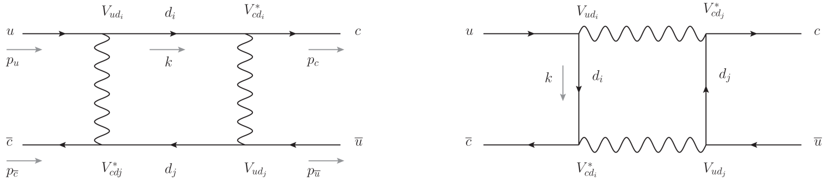
$$\Delta m_D^{\rm disp} \sim -\frac{1}{\pi}\log\frac{m_D^2}{\Lambda^2_{\rm QCD}}\left[\Gamma_{K^+\pi^-} + \Gamma_{K^-\pi^+} + \ldots\right] \sim 5\times 10^{-14}\,\mathrm{GeV}$$

$\Gamma_D \approx 1.6\times 10^{-12}\,\mathrm{GeV} \quad \Rightarrow$ So we expect $x \sim O(\%)$.

Estimation of D-$\overline{D}$-mixing in SM

Toy Problem

Just for fun: make $m_c \ll m_s \ll m_W$. Calculate



Can ignore external momenta → loop integral becomes very simple. We obtain

$$i\mathcal{M} = \frac{3g^4}{32\pi^2} \sin^2\theta_c \cos^2\theta_c \frac{m_s^2}{m_W^4} (\overline{u}_c \gamma^\mu P_L u_u)(\overline{v}_{\overline{c}} \gamma_\mu P_L v_{\overline{u}})$$

We also show the result vanishes in $SU(3)$ limit.

It is useful to classify decays based the amount of "flavor violation" required for it to proceed:

- **Cabibbo-Favored (CF)** decays involve only diagonal elements of V_{CKM}
  E.g. $A(D^0 \to K^- \pi^+) \propto V_{cs} V_{ud}^*$.
- **Singly-Cabibbo-Suppressed (SCS)** decays involve one off-diagonal CKM-matrix element
  E.g. $A(D \to K^+ K^-) \propto V_{cs} V_{us}^*$.
- **Doubly-Cabibbo-Suppressed (DCS)** decays involve two off-diagonal CKM-matrix elements
  E.g. $A(\overline{D}^0 \to K^- \pi^+) \propto V_{us} V_{cd}^*$.

- RS $A(\overline{D}^0 \to K^+\ell^-\nu)$ is CF
- WS $A(D^0 \to K^+\ell^-\nu) = 0$
- Hence $D^0 \to K^+\ell^-\nu$ can only occur via mixing!

$$\implies \quad r(t) = \left|\frac{q}{p}\right|^2 |g_-(t)|^2 \approx \frac{e^{-\Gamma t}}{4}\left|\frac{q}{p}\right|^2 (x^2+y^2)(\Gamma t)^2$$

- Theoretically clean: $\pi_s^+\ell^-$ unambiguous mixing signal.
- Disadvantages:
  - Can't measure x, y independently
  - Neutrino makes FS measurement complicated
  - rate $\propto$ (mix)^2 = TINY
- Need enhancement of mixing signature...

- Observe the WS decay:

$$\begin{aligned} \text{Amplitude} &= D^0 \xrightarrow{\text{DCS}} K^+\pi^- && \text{(small)} \\ &+ \\ &D^0 \xrightarrow{\text{mix}} \overline{D}^0 \xrightarrow{\text{CF}} K^+\pi^- && \text{(tiny)} \end{aligned}$$

- Mixing term does not get drowned out, but (DCS) $>$ (mix), so you do get an enhancement!
- Compare this to WS SLFS: $(\text{mix})^2 < (\text{mix})(\text{DCS})$.

Measure time-dependence of WS decay rate:

$$(\text{rate})^2 = e^{-\Gamma t}\left[(\text{direct})^2 + (\text{direct})(\text{via mix})(\Gamma t) + (\text{via mix})^2(\Gamma t)^2\right]$$

to determine

$$x', y', |q/p| \text{ and } \phi$$

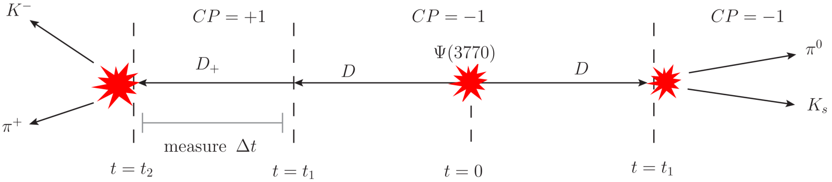
where $x'_{K\pi} = x\cos\delta_{K\pi} + y\sin\delta_{K\pi} \qquad y'_{K\pi} = y\cos\delta_{K\pi} - x\sin\delta_{K\pi}$

- This was done at BaBar, BELLE, CDF in 2007!

$\Rightarrow$ small mixing, no CPV found

- Still need strong phase $\delta_{K\pi}$ ...

CESR produces $\Psi(3770)$ charmonium on resonance in e^+e^- collisions:



Allows measurements of strong phase:

$$\cos \delta_{K\pi} = \frac{1}{2r_{K\pi}} \frac{B(D^0_+ \to K^-\pi^+) - B(D^0_- \to K^-\pi^+)}{B(D^0_+ \to K^-\pi^+) + B(D^0_- \to K^-\pi^+)}$$

$$
\begin{aligned}
x &= \left(9.72^{+2.71}_{-2.91}\right) \times 10^{-3} \\
y &= \left(7.8^{+1.8}_{-1.9}\right) \times 10^{-3} \\
\left|\frac{q}{p}\right| &= 0.86 \pm 0.31 \\
\cos\delta_{K^+\pi^-} &= 1.03^{+0.32}_{-0.18}
\end{aligned}
$$

Conclusion

- D-mixing is only portal into FCNCs in up-sector.
- Mixing has been unequivocally discovered.
- Consistent with no CPV, but that could change.
- No NP signal yet.
- Improved calculational techniques (lattice) would help.